The fluctuational region on the phase diagram of lattice Weinberg - Salam model

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Abstract

The lattice Weinberg - Salam model without fermions is investigated numerically for the realistic choice of bare coupling constants correspondent to the value of the Weinberg angle $\theta_W \sim 30^o$, and the fine structure constant $\alpha \sim \frac{1}{100}$. On the phase diagram there exists the vicinity of the phase transition between the physical Higgs phase and the unphysical symmetric phase, where the fluctuations of the scalar field become strong. The classical Nambu monopole can be considered as an embryo of the unphysical symmetric phase within the physical phase. In the fluctuational region quantum Nambu monopoles are dense and, therefore, the perturbation expansion around trivial vacuum cannot be applied. The maximal value of the cutoff at the given values of coupling constants calculated using the lattices of sizes $8^3 \times 16$, $12^3 \times 16$, and 16^4 is $\Lambda_c \sim 1.4 \pm 0.2$ Tev. As the lattice sizes used are rather small we consider this result as preliminary.

1 Introduction

In some phenomenological models that describe condensed matter systems[1] there exists the vicinity of the finite temperature phase transition that is called fluctuational region. In this region the fluctuations of the order parameter become strong. The contribution of these fluctuations to certain

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[1]One of the examples of such models is the Ginzburg - Landau theory of superconductivity.

physical observables becomes larger than the tree level estimate. Thus the perturbation theory in these models fails down within the fluctuational region.

Our main supposition is that the lattice Weinberg - Salam model (at $T = 0$) looks similar to the mentioned models. Namely, we expect that there exists the vicinity of the phase transition between the Higgs phase and the symmetric phase in the Weinberg - Salam model, where the fluctuations of the scalar field become strong and the perturbation expansion around trivial vacuum cannot be applied. According to the numerical results the continuum theory is to be approached within the vicinity of the phase transition, i.e. the cutoff is increased along the line of constant physics when one approaches the point of the transition. That's why we expect that the conventional prediction on the value of the cutoff admitted in the Standard Model based on the perturbation theory may be incorrect.

According to the conventional point of view the upper bound Λ on the cutoff in the Electroweak theory (without fermions) depends on the Higgs mass. It is decreased when the Higgs mass is increased. And at the Higgs mass around 1 Tev Λ becomes of the order of M_H. At the same time for $M_H \sim 200$ Gev the value of Λ can be made almost infinite[2]. This conclusion is made basing on the perturbation expansion around trivial vacuum.

In the present paper we report the results of the numerical investigation of the model at the value of the scalar self coupling $\lambda = 0.009$, the bare Weinberg angle $\theta_W = 30^o$, and the renormalized fine structure constant around $1/100$. The bare value of the Higgs boson mass is around 270 Gev in the vicinity of the phase transition.

We calculate the constraint effective potential $V(|\Phi|)$ for the Higgs field Φ. In the physical Higgs phase this potential has a minimum at a certain nonzero value ϕ_m of $|\Phi|$. This shows that the spontaneous breakdown of the Electroweak symmetry takes place as it should. However, there exists the vicinity of the phase transition, where the fluctuations of the Higgs field are of the order of ϕ_m while the hight of the "potential barrier"[3] $H = V(0) - V(\phi_m)$

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[2]Here we do not consider vacuum stability bound on the Higgs mass related to the fermion loops.

[3]The meaning of the words "potential barrier" here is different from that of the one - dimensional quantum mechanics as here different minima of the potential form the three - dimensional sphere while in usual $1D$ quantum mechanics with the similar potential there are two separated minima with the potential barrier between them. Nevertheless we feel it appropriate to use the chosen terminology as the value of the "potential barrier hight"

is of the order of $V(\phi_m + \delta\phi) - V(\phi_m)$, where $\delta\phi$ is the fluctuation of $|\Phi|$. We expect that in this region the perturbation expansion around trivial vacuum $\Phi = (\phi_m, 0)^T$ cannot be applied. We call this region of the phase diagram the fluctuational region (FR) in analogy to the condensed matter systems.

The mentioned supposition is confirmed by the investigation of the topological defects composed of the lattice gauge fields that are to be identified with quantum Nambu monopoles [1, 2, 3]. We show that their lattice density increases when the phase transition point is approached. Within the FR these objects are so dense that it is not possible at all to speak of them as of single monopoles [4]. Namely, within this region the average distance between the Nambu monopoles is of the order of their size. Such complicated configurations obviously have nothing to do with the conventional vacuum used of the continuum perturbation theory.

We have estimated the maximal value of the cutoff in the vicinity of the transition point. The obtained value of the cutoff appears to be around 1.4 Tev.

2   The lattice model under investigation

The lattice Weinberg - Salam Model without fermions contains gauge field $\mathcal{U} = (U, \theta)$ (where $U \in SU(2)$, $e^{i\theta} \in U(1)$ are realized as link variables), and the scalar doublet Φ_α, $(\alpha = 1, 2)$ defined on sites.

The action is taken in the form

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\begin{aligned}
S \;=\; & \beta \sum_{\text{plaquettes}} \left((1 - \tfrac{1}{2}\operatorname{Tr} U_p) + \frac{1}{\operatorname{tg}^2\theta_W}(1 - \cos\theta_p) \right) + \\
& -\gamma \sum_{xy} Re(\Phi^+ U_{xy} e^{i\theta_{xy}} \Phi) + \sum_x (|\Phi_x|^2 + \lambda(|\Phi_x|^2 - 1)^2),
\end{aligned} \qquad (1)
$$

where the plaquette variables are defined as $U_p = U_{xy}U_{yz}U^*_{wz}U^*_{xw}$, and $\theta_p = \theta_{xy} + \theta_{yz} - \theta_{wz} - \theta_{xw}$ for the plaquette composed of the vertices x, y, z, w.

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measures the difference between the potentials with and without spontaneous symmetry breaking.

[4]It has been shown in [4] that at the infinite value of the scalar self coupling $\lambda = \infty$ moving along the line of constant physics we reach the point on the phase diagram where the monopole worldlines begin to percolate. This point was found to coincide roughly with the position of the transition between the physical Higgs phase and the unphysical symmetric phase of the lattice model. This transition is a crossover and the ultraviolet cutoff achieves its maximal value around 1.4 Tev at the transition point.

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Here λ is the scalar self coupling, and $\gamma = 2\kappa$, where κ corresponds to the constant used in the investigations of the $SU(2)$ gauge Higgs model. θ_W is the Weinberg angle. Bare fine structure constant α is expressed through β and θ_W as $\alpha = \frac{\mathrm{tg}^2\theta_W}{\pi\beta(1+\mathrm{tg}^2\theta_W)}$. In our investigation we fix bare Weinberg angle equal to 30°. The renormalized fine structure constant can be extracted through the potential for the infinitely heavy external charged particles.

All simulations were performed on lattices of sizes $8^3 \times 16$ and $12^3 \times 16$. Several points were checked using the larger lattice (16^4).

In order to simulate the system we used Metropolis algorithm. The acceptance rate is kept around 0.5 via the automatical self - tuning of the suggested distribution of the fields. At each step of the suggestion the random value is added to the old value of the scalar field while the old value of Gauge field is multiplied by random $SU(2) \otimes U(1)$ matrix. We use Gaussian distribution both for the random value added to the scalar field and the parameters of the random matrix multiplied by the lattice Gauge field. We use two independent parameters for these distributions: one for the Gauge fields and another for the scalar field. The program code has been tested for the case of frozen scalar field. And the results of the papers [2] are repeated. We also have tested our code for the $U(1)$ field frozen and repeat the results of [6]. For the values of couplings used on the lattice 16^4 the autocorrelation time for the gauge fields is estimated as about $N_{auto}^g \sim 2500$ Metropolis steps. (The correlation between the values of the gauge field is less than 3% for the configurations separated by N_{auto}^g Metropolis steps. Each metropolis step consists of the renewing the fields over all the lattice.) The autocorrelation time for the scalar field is much less $N_{auto}^\phi \sim 20$. The estimated time for the preparing the equilibrium starting from the cold start is about 18000 Metropolis steps.

3 Phase diagram

In the three - dimensional (β, γ, λ) phase diagram the transition surfaces are two - dimensional. The lines of constant physics on the tree level are the lines $(\frac{\lambda}{\gamma^2} = \frac{1}{8\beta}\frac{M_H^2}{M_W^2} = \mathrm{const}; \beta = \frac{1}{4\pi\alpha} = \mathrm{const})$. We suppose that in the vicinity of the transition the deviation of the lines of constant physics from the tree level estimate may be significant. However, qualitatively their behavior is the same. Namely, the cutoff is increased along the line of constant physics when γ is decreased and the maximal value of the cutoff is achieved at the
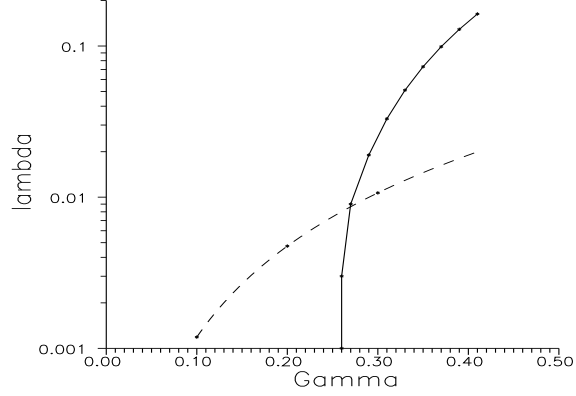
Figure 1: The phase diagram of the model in the (γ, λ)-plane at $\beta = 12$. The dashed line is the tree - level estimate for the line of constant physics correspondent to bare $M_H^0 = 270$ Gev. The continuous line is the line of phase transition between the physical Higgs phase and the unphysical symmetric phase.

transition point. Nambu monopole density in lattice units is also increased when the ultraviolet cutoff is increased.

At $\beta = 12$ the phase diagram is represented on Figure 1. The physical Higgs phase is situated right to the transition line. The position of the transition is localized at the point where the susceptibility extracted from the Higgs field creation operator achieves its maximum. The following variable is considered as creating the Z boson:

$$Z_{xy} = Z_x^{\mu} = \sin\left[\mathrm{Arg}(\Phi_x^+ U_{xy} e^{i\theta_{xy}} \Phi_y)\right]. \tag{2}$$

We use the susceptibility $\chi = \langle H^2 \rangle - \langle H \rangle^2$ extracted from $H = \sum_y Z_{xy}^2$. We observe no difference between the values of the susceptibility calculated using the lattices of the sizes $8^3 \times 16$, $12^3 \times 16$, and 16^4. This indicates that the transition may be a crossover.

It is worth mentioning that the value of the renormalized Higgs boson mass does not deviate significantly from its bare value. Namely, for λ around 0.009 and γ in the vicinity of the phase transition bare value of the Higgs mass is around 270 Gev while the observed renormalized value is 300 ± 70 Gev (see the next section for the details).

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4 Masses and the lattice spacing

In order to evaluate the masses of the Z-boson and the Higgs boson we use the correlators:

$$\frac{1}{N^6}\sum_{\bar{x},\bar{y}}\langle\sum_{\mu}Z_x^{\mu}Z_y^{\mu}\rangle \sim e^{-M_Z|x_0-y_0|} + e^{-M_Z(L-|x_0-y_0|)} \qquad (3)$$

and

$$\frac{1}{N^6}\sum_{\bar{x},\bar{y}}(\langle H_xH_y\rangle - \langle H\rangle^2) \sim e^{-M_H|x_0-y_0|} + e^{-M_H(L-|x_0-y_0|)}, \qquad (4)$$

Here the summation $\sum_{\bar{x},\bar{y}}$ is over the three "space" components of the four - vectors x and y while x_0, y_0 denote their "time" components. N is the lattice length in "space" direction. L is the lattice length in the "time" direction.

In lattice calculations we used two different operators that create Higgs bosons: $H_x = |\Phi|$ and $H_x = \sum_y Z_{xy}^2$. In both cases H_x is defined at the site x, the sum $\sum_y$ is over its neighboring sites y.

After fixing the unitary gauge ($\Phi_2 = 0$; $\Phi_1 \in \mathcal{R}$; $\Phi_1 \geq 0$), lattice Electroweak theory becomes a lattice $U(1)$ gauge theory with the $U(1)$ gauge field

$$A_{xy} = A_x^{\mu} = [-Z' + 2\theta_{xy}]\,\mathrm{mod}\,2\pi, \qquad (5)$$

where the new lattice Z - boson field (different from (2)) is defined as

$$Z' = \mathrm{Arg}(\Phi_x^+ U_{xy}e^{i\theta_{xy}}\Phi_y). \qquad (6)$$

The usual Electromagnetic field is related to A as $A_{\mathrm{EM}} = A + Z' - 2\sin^2\theta_W Z'$.

The physical scale is given in our lattice theory by the value of the Z-boson mass $M_Z^{phys} \sim 91$ GeV. Therefore the lattice spacing is evaluated to be $a \sim [91\mathrm{GeV}]^{-1}M_Z$, where M_Z is the Z boson mass in lattice units. The similar calculations have been performed in [4] for $\lambda = \infty$. It has been shown that the W - boson mass unlike M_Z depends strongly on the lattice size due the photon cloud. Therefore the Z - boson mass was used in [4] in order to fix the scale. That's why in the present paper we do not consider the W - boson mass.

Our data obtained on the lattice $8^3 \times 16$ shows that $\Lambda = \frac{\pi}{a} = (\pi \times 91 \text{ GeV})/M_Z$ is increased slowly with the decrease of γ at any fixed λ. We investigated carefully the vicinity of the transition point at fixed $\lambda = 0.009$ and $\beta = 12$. It has been found that at the transition point $\gamma_c = 0.273 \pm 0.002$

the value of Λ is equal to 1.4 ± 0.2 Tev. The check of a larger lattice (of size $12^3 \times 16$) does not show an essential increase of this value. We also calculate Λ on the lattice 16^4 at the two points (one is at the transition point and another is within the physical phase). Again we do not observe the increase of Λ. However, at the present moment we do not exclude that such an increase can be observed on the larger lattices. That's why careful investigation of the dependence of Λ on the lattice size (as well as on λ) must be performed in order to draw the final conclusion. On Fig. 3 the dependence of M_Z in lattice units on γ is represented at $\lambda = 0.009$ and $\beta = 12$.

In the Higgs channel the situation is much more difficult. First, due to the lack of statistics we cannot estimate the masses in this channel using the correlators (4) at all considered values of γ. At the present moment we can represent the data at the two points on the lattice $8^3 \times 16$: ($\gamma = 0.274$, $\lambda = 0.009$, $\beta = 12$) and ($\gamma = 0.290$, $\lambda = 0.009$, $\beta = 12$). The first point roughly corresponds to the position of the transition while the second point is situated deep within the Higgs phase. The sets of coupling chosen correspond to bare Higgs mass around 270 Gev. That's why in this channel, in principle, bound states of the gauge bosons may appear. This situation was already considered in earlier studies of $SU(2)$ Gauge - Higgs model (see, for example, [7] and references therein). Following these studies we interpret the mass found in this channel at small "time" separations as the Higgs mass. We suppose that the bound states of gauge bosons may appear in correlator (4) at larger "time" separations.

At the point ($\gamma = 0.274$, $\lambda = 0.009$, $\beta = 12$) we have collected enough statistics to calculate correlator (4) up to the "time" separation $|x_0 - y_0| = 4$. The value $\gamma = 0.274$ corresponds roughly to the position of the phase transition. The mass found in this channel in lattice units is $M_H^L = 0.75 \pm 0.1$ while bare value of M_H is $M_H^0 \sim 270$ Gev. At the same time $M_Z^L = 0.23 \pm 0.007$. Thus we estimate at this point $M_H = 300 \pm 40$ Gev.

At the point ($\gamma = 0.29$, $\lambda = 0.009$, $\beta = 12$) we calculate the correlator with reasonable accuracy up to $|x_0 - y_0| = 3$. At this point bare value of M_H is $M_H^0 \sim 260$ Gev while the renormalized Higgs mass in lattice units is $M_H^L = 1.2 \pm 0.3$. At the same time $M_Z^L = 0.41 \pm 0.01$. Thus we estimate at this point $M_H = 265 \pm 70$ Gev.

It is worth mentioning that in order to calculate Z - boson mass we fit the correlator (3) for $8 \geq |x_0 - y_0| \geq 1$. In order to calculate the Higgs boson mass at $\gamma = 0.274$ we use the data for the correlator (4) at $4 \geq |x_0 - y_0| \geq 0$. In order to calculate the Higgs boson mass at $\gamma = 0.29$ we use the correlator
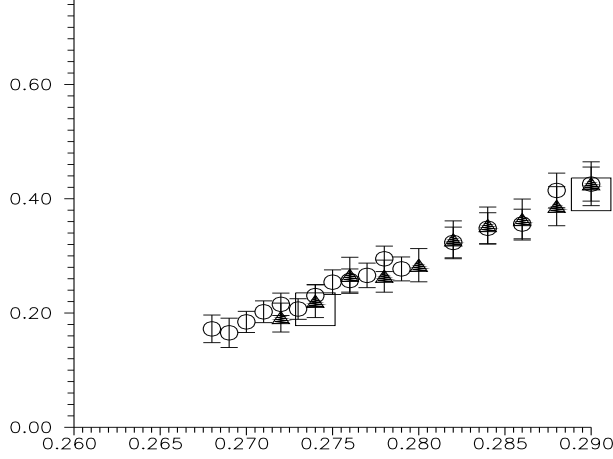
Figure 2: Z - boson mass in lattice units at $\lambda = 0.009$ and $\beta = 12$. Circles correspond to lattice $8^3 \times 16$. Triangles correspond to lattice $12^3 \times 16$. Squares correspond to lattice 16^4 (the error bars are about of the same size as the symbols used).

for $3 \geq |x_0 - y_0| \geq 0$.

5 Effective constraint potential

We have calculated the constraint effective potential for $|\Phi|$ using the histogram method. The calculations have been performed on the lattice $8^3 \times 16$. The probability $h(\phi)$ to find the value of $|\Phi|$ within the interval $[\phi - 0.05; \phi + 0.05)$ has been calculated for $\phi = 0.05 + N * 0.1$, $N = 0, 1, 2, ...$ This probability is related to the effective potential as $h(\phi) = \phi^3 e^{-V(\phi)}$. That's why we extract the potential from $h(\phi)$ as

$$V(\phi) = -\log h(\phi) + 3 \log \phi \tag{7}$$

It is worth mentioning that $h(0.05)$ is calculated as the probability to find the value of $|\Phi|$ within the interval $[0; 0.1]$. Within this interval $\log \phi$ is ill defined. That's why we exclude the point $\phi = 0.05$ from our data. Instead we calculate $V(0)$ using the extrapolation of the data at $0.15 \leq \phi \leq 2.0$. The extrapolation is performed using the polynomial fit with the powers of ϕ up to the third (average deviation of the fit from the data is around 1 per
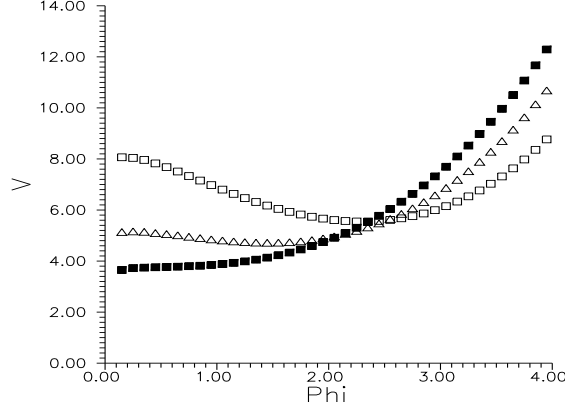
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Figure 3: The effective constraint potential at $\lambda = 0.009$ and $\beta = 12$. Black squares correspond to $\gamma_c = 0.273$. Empty squares correspond to $\gamma = 0.29$. Triangles correspond to $\gamma = 0.279$. The error bars are about of the same size as the symbols used.

cent). Next, we introduce the useful quantity $H = V(0) - V(\phi_m)$, which is called the potential barrier hight (here ϕ_m is the point, where V achieves its minimum).

In Table 1 we represent the values of ϕ_m and H for $\lambda = 0.009$, $\beta = 12$. One can see that the values of ϕ_m and H increase when γ is increased. At $\gamma = 0.273$ the minimum of the potential is at $\phi = 0$. This point corresponds to the maximum of the susceptibility constructed of the Higgs field creation operator. At $\gamma = 0.274$ we also observe the only minimum for the potential at $\phi = 0$. At $\gamma = 0.275$ minimum of the potential is observed at $\phi_m = 0.85 \pm 0.1$ with the very small barrier hight. That's why we localize the position of the transition point at $\gamma = 0.273 \pm 0.002$.

It is important to understand which value of barrier hight can be considered as small and which value can be considered as large. Our suggestion is to compare $H = V(0) - V(\phi_m)$ with $H_{\text{fluct}} = V(\phi_m + \delta\phi) - V(\phi_m)$, where $\delta\phi$ is the fluctuation of $|\Phi|$.

From Table 1 it is clear that there exists the value of γ (we denote it γ_{c2}) such that at $\gamma_c < \gamma < \gamma_{c2}$ the barrier hight H is of the order of H_{fluct} while for $\gamma_{c2} << \gamma$ the barrier hight is essentially larger than H_{fluct}. The rough estimate for this pseudocritical value is $\gamma_{c2} \sim 0.278$.

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Table 1: The values of ϕ_m, H, H_{fluct}, and Nambu monopole density ρ at selected values of γ for $\lambda = 0.009$, $\beta = 12$ (Lattice $8^3 \times 16$.)

γ	ϕ_m	H	H_{fluct}	ρ
0.273	0	0	0.1 ± 0.1	0.098 ± 0.001
0.274	0	0	0.04 ± 0.1	0.081 ± 0.001
0.275	0.85 ± 0.1	0.01 ± 0.06	0.15 ± 0.05	0.067 ± 0.001
0.276	1.05 ± 0.1	0.05 ± 0.06	0.16 ± 0.01	0.054 ± 0.001
0.277	1.25 ± 0.05	0.19 ± 0.05	0.25 ± 0.05	0.044 ± 0.001
0.278	1.35 ± 0.1	0.28 ± 0.07	0.25 ± 0.06	0.035 ± 0.001
0.279	1.45 ± 0.05	0.5 ± 0.06	0.25 ± 0.06	0.028 ± 0.001
0.282	1.75 ± 0.05	1.04 ± 0.07	0.31 ± 0.07	0.014 ± 0.001
0.284	1.95 ± 0.05	1.41 ± 0.08	0.38 ± 0.08	0.0082 ± 0.0005
0.286	2.05 ± 0.05	1.86 ± 0.08	0.35 ± 0.08	0.0049 ± 0.0002
0.288	2.15 ± 0.05	2.33 ± 0.08	0.32 ± 0.07	0.0029 ± 0.0002
0.29	2.25 ± 0.05	2.82 ± 0.08	0.44 ± 0.08	0.0017 ± 0.0001

We estimate the fluctuations of $|\Phi|$ to be around $\delta\phi \sim 0.6$ for all considered values of γ at $\lambda = 0.009$, $\beta = 12$. It follows from our data that $\phi_m >> \delta\phi$ at $\gamma_{c2} << \gamma$ while $\phi_m \sim \delta\phi$ at $\gamma_{c2} > \gamma$.

Basing on these observations we expect that in the region $\gamma_{c2} << \gamma$ the usual perturbation expansion around trivial vacuum of spontaneously broken theory can be applied to the lattice Weinberg - Salam model while in the FR $\gamma_c < \gamma < \gamma_{c2}$ it cannot be applied.

At the value of γ equal to $\gamma_{c2} \sim 0.278$ the calculated value of the cutoff is 1.0 ± 0.1 Tev.

6   The renormalized coupling

In order to calculate the renormalized fine structure constant $\alpha_R = e^2/4\pi$ (where e is the electric charge) we use the potential for infinitely heavy external fermions. We consider Wilson loops for the right-handed external leptons: $\mathcal{W}_{\text{lept}}^{\text{R}}(l) = \langle \text{Re} \, \Pi_{(xy)\in l} e^{2i\theta_{xy}} \rangle$. Here l denotes a closed contour on the lattice. We consider the following quantity constructed from the rectangular Wilson loop of size $r \times t$: $\mathcal{V}(r) = \lim_{t\to\infty} \log \frac{\mathcal{W}(r\times t)}{\mathcal{W}(r\times(t+1))}$. At large enough distances we

expect the appearance of the Coulomb interaction $\mathcal{V}(r) = -\frac{\alpha_R}{r} + const.$

At $\lambda = 0.009$, $\beta = 12$, $\gamma = \gamma_c(\lambda) \sim 0.273$ the renormalized fine structure constant calculated on the lattice $8^3 \times 16$ is $\alpha_R = \frac{1}{97\pm2}$. For $0.27 \leq \gamma \leq 0.29$ the value of α_R varies between $\frac{1}{97\pm2}$ and $\frac{1}{100\pm2}$. These values coincide within the statistical errors with the values calculated on the larger lattice ($12^3 \times 16$). This indicates that the value of α_R does not depend on the lattice size also for the small values of λ. The calculated values are to be compared with bare constant $\alpha_0 = 1/(4\pi\beta)$.

7   Nambu monopoles

In this section we remind the reader what is called Nambu monopole [1, 2]. First let us define the continuum Electroweak fields as they appear in the Weinberg-Salam model. The continuum scalar doublet is denoted as Φ. After fixing the unitary gauge $\Phi_1 \in \mathcal{R}$, $\Phi_2 = 0$, the Z-boson field Z^μ and electromagnetic field A_{EM}^μ are defined as $Z^\mu = \frac{1}{2}\text{Tr}C^\mu\sigma_3 + B^\mu$, and $A_{\text{EM}}^\mu = 2B^\mu - 2\sin^2\theta_W Z^\mu$, where C^μ and B^μ are the corresponding $SU(2)$ and $U(1)$ gauge fields of the Standard Model.

Nambu monopoles are defined as the endpoints of the Z-string [1]. The Z-string is the classical field configuration that represents the object, which is characterized by the magnetic flux extracted from the Z-boson field. Namely, for a small contour $\mathcal{C}$ winding around the Z - string one should have

$$\int_{\mathcal{C}} Z^\mu dx^\mu \sim 2\pi; \int_{\mathcal{C}} A_{\text{EM}}^\mu dx^\mu \sim 0; \int_{\mathcal{C}} B^\mu dx^\mu \sim 2\pi\sin^2\theta_W. \qquad (8)$$

The string terminates at the position of the Nambu monopole. The hypercharge flux is supposed to be conserved at that point. Therefore, a Nambu monopole carries electromagnetic flux $4\pi\sin^2\theta_W$. The size of Nambu monopoles was estimated [1] to be of the order of the inverse Higgs mass, while its mass should be of the order of a few TeV. According to [1] Nambu monopoles may appear only in the form of a bound state of a monopole-antimonopole pair.

In lattice theory the classical solution corresponding to a Z-string should be formed around the 2-dimensional topological defect which is represented by the integer-valued field defined on the dual lattice $\Sigma = \frac{1}{2\pi}{}^*([dZ']_{\text{mod}2\pi} - dZ')$. (Here we used the notations of differential forms on the lattice. For a definition of those notations see, for example, [5]. Lattice field Z' is defined

in Eg. (6).) Therefore, Σ can be treated as the worldsheet of a *quantum Z*-string[3, 2]. Then, the worldlines of quantum Nambu monopoles appear as the boundary of the *Z*-string worldsheet: $j_Z = \delta\Sigma$.

It has been mentioned in section 5 that our lattice model becomes $U(1)$ gauge model after fixing the unitary gauge. The corresponding compact $U(1)$ gauge field is given by Eq. (5). Therefore one may try to extract monopole trajectories directly from A. The monopole current is given by

$$j_A = \frac{1}{2\pi}{}^*d([dA]\mathrm{mod}2\pi) \qquad (9)$$

Eg. (5) in continuum notations is

$$A^\mu = -Z^\mu + 2B^\mu, \qquad (10)$$

where B is the hypercharge field. Its strength is divergenceless. As a result in continuum theory the net Z flux emanating from the center of the monopole is equal to the net A flux with the opposite sign. (Both A and Z are undefined inside the monopole.) This means that in the continuum limit the position of the Nambu monopole must coincide with the position of the antimonopole extracted from the field A. Therefore, one can consider Eq. (9) as another definition of a quantum Nambu monopole [2, 4]. Actually, in our numerical simulations we use the definition of Eq. (9).

8 Nambu monopole density

According to the previous section the worldlines of the quantum Nambu monopoles can be extracted from the field configurations according to Eq. (9). The monopole density is defined as $\rho = \left\langle \frac{\sum_{\text{links}} |j_{\text{link}}|}{4V^L} \right\rangle$, where V^L is the lattice volume.

In Table 1 we represent Nambu monopole density as a function of γ at $\lambda = 0.009$, $\beta = 12$. The value of monopole density at $\gamma_c = 0.273$ is around 0.1. At this point the value of the cutoff is $\Lambda \sim 1.4 \pm 0.2$ Tev.

According to the classical picture the Nambu monopole size is of the order of M_H^{-1}. Therefore for $a^{-1} \sim 430$ Gev and $M_H \sim 300$ Gev the expected size of the monopole is about 1.4 lattice units.

The monopole density around 0.1 means that among 10 sites there exists 4 sites that are occupied by the monopole. Average distance between the two

monopoles is, therefore, less than 1 lattice spacing and it is not possible at all to speak of the given configurations as of representing the physical Nambu monopole.

At $\gamma = \gamma_{c2} \sim 0.278$ the Nambu monopole density is around 0.035. This means that among 7 sites there exists one site that is occupied by the monopole. Average distance between the two monopoles is, therefore, approximately 2 lattice spacings or $\sim \frac{1}{160\,\text{Gev}}$. Thus, the Nambu monopole density in physical units is around $[160\,\text{Gev}]^3$. We see that at this value of γ the average distance between Nambu monopoles is of the order of their size.

We summarize the above observations as follows. Within the fluctuational region the configurations under consideration do not represent single Nambu monopoles. Instead these configurations can be considered as the collection of monopole - like objects that is so dense that the average distance between the objects is of the order of their size. On the other hand, at $\gamma >> \gamma_{c2}$ the considered configurations do represent single Nambu monopoles and the average distance between them is much larger than their size. In other words out of the FR vacuum can be treated as a gas of Nambu monopoles while within the FR vacuum can be treated as a liquid composed of monopole - like objects.

It is worth mentioning that somewhere inside the Z string connecting the classical Nambu monopoles the Higgs field is zero: $|\Phi| = 0$. This means that the Z string with the Nambu monopoles at its ends can be considered as an embryo of the symmetric phase within the Higgs phase. We observe that the density of these embryos is increased when the phase transition is approached. Within the fluctuational region the two phases are mixed, which is related to the large value of Nambu monopole density.

That's why we come to the conclusion that vacuum of lattice Weinberg - Salam model within the FR has nothing to do with the continuum perturbation theory. This means that the usual perturbation expansion around trivial vacuum (gauge field equal to zero, the scalar field equal to $(\phi_m, 0)^T$) cannot be valid within the FR. This might explain why we do not observe in our numerical simulations the large values of Λ predicted by the conventional perturbation theory.

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9 Conclusions

In the present paper we demonstrate that there exists the so - called fluctuational region (FR) on the phase diagram of the lattice Weinberg - Salam model. This region is situated in the vicinity of the phase transition between the physical Higgs phase and the unphysical symmetric phase of the model. We calculate the effective constraint potential $V(\phi)$ for the Higgs field. It has a minimum at the nonzero value ϕ_m in the physical Higgs phase. Within the FR the fluctuations of the scalar field become of the order of ϕ_m. Moreover, the barrier hight $H = V(0) - V(\phi_m)$ is of the order of $V(\phi_m + \delta\phi) - V(\phi_m)$, where $\delta\phi$ is the fluctuation of $|\Phi|$.

The scalar field must be equal to zero somewhere within the classical Nambu monopole. That's why this object can be considered as an embryo of the unphysical symmetric phase within the physical Higgs phase of the model. We investigate properties of the quantum Nambu monopoles. Within the FR they are so dense that the average distance between them becomes of the order of their size. This means that the two phases are mixed within the FR.

All these results show that the vacuum of lattice Weinberg - Salam model in the FR is essentially different from the trivial vacuum used in the conventional perturbation theory. As a result the usual perturbation theory cannot be applied in this region.

It is important that the continuum physics is to be approached in the vicinity of the mentioned phase transition. The ultraviolet cutoff is increased when the transition point is approached along the line of constant physics. Our numerical results show that at M_H around 300 Gev and the fine structure constant around 1/100 the maximal value of the cutoff admitted out of the FR for the considered lattice sizes cannot exceed the value around 1.0 ± 0.1 Tev. Within the FR the larger values of the cutoff can be achieved in principle although we expect the correspondent results cannot be related directly to the real continuum physics. The absolute maximum for the value of the cutoff within the Higgs phase of the lattice model is achieved at the point of the phase transition. Our estimate for this value is 1.4 ± 0.2 Tev for the lattice sizes $8^3 \times 16$, $12^3 \times 16$, and 16^4. Out of the fluctuational region the behavior of the lattice model in general is close to what we expect basing on the continuous perturbation theory.

We do not observe the dependence of the observables on the lattice size ($8^3 \times 16$, $12^3 \times 16$, and 16^4). In particular, the susceptibility extracted from

the Higgs field creation operator does not depend on the lattice size. This indicates that the transition at the considered value of $\lambda = 0.009$ is a crossover. However, our results have been obtained on rather small lattices. That's why careful investigation of the dependence of the observables on the lattice size (as well as on the Higgs mass) must be performed in order to draw the final conclusions. The inclusion of the dynamical fermions into the consideration may also be important.

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